On Jump Processes in the Foreign Exchange and Stock Markets

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This article investigates the existence of discontinuities in the sample path of exchange rates and of a stock market index. Maximum-likelihood estimation of a mixed jump-diffusion process reveals that exchange rates exhibit systematic discontinuities, even after allowingfor conditional heteroskedasticity in the diffusion process. The results are much more significant in the foreign exchange market than in the stock market, which suggests differences in the structure of these markets. Finally, this jump component is shown to explain some of the empirically observed mispricings in the currency options market.

The objective of this article is to analyze and compare the empirical distribution of returns in the stock market and in the foreign exchange market. There are a number of reasons why a better understanding of the stochastic processes driving prices in these markets would be useful. Many financial models rely heavily on the assumption of a particular stochastic process, while relatively little attention is paid to the empirical fit of the postulated distribution. As a result, models like option pricing models are applied indiscriminately to various markets such as the stock market and the foreign exchange market when the underlying processes may be fundamentally different. The foreign exchange market, for instance, is characterized by active exchange rate man-

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agement policies that do not have counterparts in the stock market. A systematic comparison of these two markets, à la Frenkel and Mussa (1980), could reveal interesting insights into the behavior of stock prices and exchange rates.

This article compares the empirical fit of two classes of distributions as alternatives to the usual continuous diffusion process. First, it is important to examine if discontinuities are present in the data, given that many continuous-time models are based on the assumption that asset prices have continuous sample paths. Evidence of discontinuities would indicate that one of the basic building blocks of many financial models is inconsistent with the data. Recently, Jarrow and Rosenfeld (1984) and Ball and Torous (1985) have found evidence that *daily* stock returns are characterized by lognormally distributed jumps, which indicates that the assumption of stationary lognormal processes may be suspect for common stocks. These discontinuities are not apparent in monthly and weekly data. Whether these results carry to the foreign exchange market is an open question.

The findings of discontinuities, however, may not be quite unexpected given the observed leptokurtosis in the distribution of exchange rates. In fact, any process that can generate “fat” tails could potentially lead to the rejection of the stationary diffusion process against the postulated alternative. The rejection of the diffusion process may be due to the assumption of stationary parameters instead of the existence of discontinuities. Because diffusion processes with time-varying parameters are consistent with continuous-time models, the question is whether any observed jump process disappears when specific allowance is made for time-varying parameters. As a first approach, this article considers a tractable specification of time-varying second moments: the autoregressive conditional heteroskedastic (ARCH) model, first proposed by Engle (1982). This article employs maximum-likelihood estimation, which allows formal tests of the fit of various models through nested hypotheses.

Finally, the article attempts to address the question of economic versus statistical significance by illustrating the implications of the results for the pricing of options. Ball and Torous (1985) report that the existence of jumps in common stock returns leads to relatively small deviations between the Black-Scholes and Merton option valuation models and thus may not be operationally significant for the stock option market. On the other hand, Bodurtha and Courtadon (1987) recently documented systematic biases in the American option pricing model applied to currency options. If jump components are relatively more important for exchange rates, these results could be explained by discontinuities in exchange rates.

This article is organized as follows. Section 1 presents a review of the issues and literature on the distribution of exchange rate movements. The methodology and models used are presented in Section 2. Section 3 analyzes the results of the maximum-likelihood estimation applied to the monthly and weekly U.S. dollar/German mark ($/DM) exchange rate, as well as to the U.S. value-weighted stock market index. The importance of these results for the pricing of foreign currency options is demonstrated in Section 4. Section 5 contains a few concluding comments.

1. Issues and Literature Review

The empirical literature on distributions in the foreign exchange market has grown largely as an extension of the abundant literature on distributions in the stock market, without much consideration to potential differences in the structure of the two markets. No study has looked at the existence of jump processes in the sample path of exchange rates. Yet this is an important topic, given its implications for the use of continuous-time models in international finance as well as for models of exchange rate determination.

There is no reason why the information arrival process should be the same in the foreign exchange market and in the stock market. During fixed-exchange-rate regimes, discontinuities obviously occur when parity values are realigned. But even with flexible exchange rates, realignments in cross-exchange rates, for example, within the European Monetary System, could be reflected in the exchange rate vis à vis the dollar. In addition, jumps in exchange rates may be generated by discontinuities in the arrival of "news," which Mussa (1979) and Frenkel (1981) argued should be the predominant cause of exchange rate movements, or by changes in monetary policies directed at affecting the external value of a currency, which Flood and Hodrick (1986) labeled "process switching."

There is some empirical evidence on the distributional characteristics of exchange rate movements, but less than for the stock market. It is a documented fact[1] that the distribution of changes in exchange rates exhibits fatter tails than would be expected from a normal distribution. Given this result, the empirical distribution could be potentially explained by one of the three possible classes of models: (1) a stationary process, such as a Paretian stable or a Student's *t* distribution, with fatter tails than the normal model, (2) a mixture of stationary distributions, such as two normal distributions with different means or variances, or a mixture of a normal and jump process, or (3) a distribution such as the normal distribution with time-varying parameters. Any of these choices could improve the fit over the normal distribution. A number of these models have been applied to the foreign exchange market, usually independently, and it is not clear which of these hopefully parsimonious representations is most appropriate. McFarland, Pettit, and Sung (1982), for example, document a day-of-the-week effect, which implies different distributional parameters, but argue that *weekly* changes in exchange rates are appropriately characterized by stable Paretian distributions. Friedman and Vandersteel (1982) and Boothe

[1] See, for instance, Farber, Roll, and Solnik (1977), Westerfield (1977), McFarland, Pettit, and Sung (1982) and Wasserfallen and Zimmermann (1985) for monthly, weekly, daily, and intradaily exchange rates, respectively.

and Glassman (1987), on the other hand, suggest that exchange rate changes can be described by normal distributions with time-dependent parameters. This is confirmed by Hsieh (1988), who shows that the distributions of daily exchange rates are characterized by time-varying parameters. The problem with these studies is that the proposed models have not been formally pitted against each other. Thus, it would be of considerable interest to compare two classes of models and formally test whether one is more appropriate than the other.

The first model considered in this study will be the jump-diffusion process, which could explain the skewness in exchange rate distributions reported by Calderon-Rossell and Ben-Horim (1982) and So (1987). A possible alternative that is consistent with continuous-time financial models is that of a diffusion process with time-varying parameters. The model chosen here is the ARCH process, introduced by Engle (1982), in which the conditional variance is a deterministic function of past data. This model was first applied to the foreign exchange market by Domowitz and Hakkio (1985). The simplest specification is a first-order model in which the conditional variance is a linear function of the past squared innovation. This could be easily extended to more general specifications. Both models will be tested and compared to each other. The question is whether discontinuities can be identified even after allowing for diffusion processes with time-varying parameters. Comparative results will be reported for both the foreign exchange market and the stock market, which may yield some insights into differences in the structures of the two markets.

2 Methodology

This section briefly presents the stochastic processes under investigation as well as the maximum-likelihood estimation procedure. Detailed derivations are contained in the Appendix. Define x_t as the logarithm of price relatives. $ln(P_t/P_{t-1})$, where P is either the dollar price of the foreign currency or the dollar price of the normalized stock market index. The assumption that prices follow the diffusion process $dP_t/P_t = \alpha\, dt + \sigma\, dz_t$ implies that $x_t \sim N(\mu, \sigma^2)$ or is normally distributed with mean $\mu \equiv \alpha - \sigma^2/2$ and variance σ^2, both defined per unit time. In discrete time,

$$\ln(P_t/P_{t-1}) = \mu + \sigma z \tag{1}$$

where z is a standard normal deviate.

Discontinuities can be modeled by the mixed jump-diffusion process $dP_t/P_t = \alpha\, dt + \sigma\, dz_t + dq_t$, in which the Poisson process dq_t is characterized by a mean number of jumps occurring per unit time λ as well as a jump size Y, which is assumed independently lognormally distributed $\ln Y \sim N(\theta, \delta^2)$. Thus x_t can be written as

$$\ln(P_t/P_{t-1}) = \mu + \sigma z + \sum_{t=1}^{n_t} \ln Y_t \tag{2}$$

where n_t is the actual number of jumps during the interval.

But the observed leptokurtosis could also be explained by a diffusion model with time-varying second moments. The alternative explored here is that of a first-order ARCH process,[2] in which the conditional variance is defined as a nonstochastic function of the last squared innovation. Conditional on information at t - 1, the distribution of x_t is given by

$$\ln(P_t/P_{t-1}) \mid t-1 = \mu + \sqrt{b_t}\, z$$
$$b_t \equiv E_{t-1}(\sigma_t^2) = \alpha_0 + \alpha_1(x_{t-1} - \mu)^2 \tag{3}$$

in which α_1 is the autoregressive parameter inducing heteroskedasticity.

The question as to whether time-varying second moments can fully account for the observed fat tails can be answered by considering a specification combining both ARCH and jump processes:

$$\ln(P_t/P_{t-1}) \mid t-1 = \mu + \sqrt{b_t}z + \sum_{i=1}^{n_t} \ln Y_i \tag{4}$$

This model can be used as an alternative against which the hypothesis of a pure ARCH process [Equation (3)] can be tested.

The parameters of interest are estimated by numerical maximization of the likelihood function of the parameter vector ϕ given the observations x, $L(\phi; x)$. The likelihood functions and the first-order conditions are derived in the Appendix. Maximum-likelihood estimation presents a number of advantages in this context.[3] The estimates are consistent and invariant, with normal-asymptotic distributions with known parameters. In addition, maximum-likelihood estimation permits formal tests of the relative fit of various distributions. *Nested* hypotheses can be tested using the *generalized* likelihood ratio $\Lambda = \sup_{\phi\in\Phi_0} L(\phi; x)/\sup_{\phi\in\Phi} L(\phi; x)$ of the maximized-likelihood functions under the null and under the enlarged parameter space Φ, which also includes the alternative hypothesis 9,. Under the null Φ_0, the statistic -2 ln Λ has a chi-square distribution with degrees of freedom equal to the difference in the number of parameters between the two models. Thus, the improvement in the maximized likelihood indicates to what extent an enlarged specification helps in fitting the data.

It should be emphasized that the alternative hypothesis must include the null hypothesis as a special case to employ this test. Results based on a *simple* likelihood ratio-defined as the ratio of the maximized likelihood for one model to the maximized likelihood for another model-are only suggestive, because they only indicate which model is more “likely” and are not formal tests of hypotheses. One model, for instance, may require estimating many more parameters than the other and thus may yield a higher value of the maximized likelihood, when it is not clear whether the improvement is due to a better functional fit or a greater number of

[2] This is the simplest model that captures the time variation in second moments. It could be extended to include autocorrelation and/or time variation in first and second moments. as in Hsieh (1988).

[3] Unfortunately, the method cannot be applied to the estimation of stable distributions, whose densities, with few exceptions, are not known in closed form.

parameters.[4] If one wants to decide between nonnested models, such as the ARCH, and the mixed jump-diffusion process, the criterion for model selection should explicitly penalize for the number of parameters. Schwarz (1978) suggested choosing the model a posteriori most probable. A simple approximation to this bayesian approach is to choose the model with the lowest value of the "Schwarz Criterion"

$$SC = -2 \ln L(\hat{\phi}; x) + (\ln T)K$$

where K is the number of parameters. Here the trade-off between precision and parsimony is clear.

3. Empirical Results

The following data sources were used in the empirical analysis. Daily observations for exchange rates were obtained from Data Resources International for the period June 1973 to December 1985. Daily stock market returns were taken from the Center for Research in Security Prices (CRSP) database, which provides a value-weighted market index of all quoted NYSE and AMEX stocks. End-of-month and end-of-week data were sampled from the daily files. Because 1973 was a year of transition from fixed to flexible exchange rates, all samples start in January 1974.[5] Given the complex distributional changes observed for different days of the week,[6] daily data will not be investigated here. The analysis focuses on monthly and weekly data, which is the data usually chosen for tests of asset pricing models and of models of exchange rate determination. For conciseness, only detailed results for the $/DM rate are presented in this article; the analysis was also performed for the British pound and the Japanese yen with similar results.

Table 1 shows summary statistics for monthly and weekly logarithmic changes in the $/DM exchange rate and in the value-weighted stock market. The departures from the normal density are apparent from the high excess kurtosis coefficients, especially for weekly data.. Normal densities imply zero coefficients of skewness and excess kurtosis. The high asymptotic t -statistics clearly reveal "fat-tailed" distributions. The pattern of autocorrelation, on the other hand, indicates little serial correlation.[7]

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[4] This procedure has been used by Rogalski and Vinso (1978), who argue that the Student's t distribution is more appropriate than the stable distribution for weekly changes in flexible exchange rates. See also Kon (1984) for tests involving stock market prices.

[5] This allows monthly and weekly returns to start at the same date and also allows the use of some presample data for the ARCH process.

[6] See, for example, McFarland, Pettit, and Sung (1982) and Hsieh (1988).

[7] Some autocorrelation coefficients for weekly stock market returns seem abnormally high. For this series, the Box-Pierce statistic also reflects the hypothesis that the correlation coefficients are jointly zero. This effect, however. can be attributed to a few observations in 1975 and is restricted to a short sample period. Hsieh (1988) also argued that rejections in such tests can be caused by Incorrect estimates of standard errors due to heteroskedasticity.

Table 1
Summary statistics for the $/DM rate and value-weighted CRSP index for the period January 1974–December 1985

	$/DM rate		Value-weighted CRSP index	
	Monthly	Weekly	Monthly	Weekly
Mean	0.000695	0.000134	0.009942	0.002277
Standard deviation	0.033505	0.014338	0.046482	0.021956
Skewness	0.0323	0.2510	0.0593	0.0702
	(0.16)	(2.56)	(0.29)	(0.72)
Excess kurtosis	1.5588*	3.291*	0.894	2.922*
	(3.82)	(16.81)	(2.19)	(14.92)
Autocorrelations ρ-lags:				
1	−0.045	0.069	0.030	−0.005
2	0.129	0.020	−0.046	−0.018
3	0.083	0.023	0.037	0.144*
4	0.006	0.044	0.052	−0.017
5	0.009	−0.058	0.140	−0.082
6	−0.053	−0.015	−0.142	0.078
7	0.042	−0.006	−0.082	−0.036
8	0.078	0.035	−0.053	−0.056
9	−0.066	0.036	−0.071	0.079
10	0.064	0.046	0.003	−0.059
11	0.065	0.035	−0.062	−0.103*
12	−0.119	0.019	0.097	0.107*
Standard error (ρ)	0.083	0.040	0.083	0.040
Box–Pierce (*P*-value)	7.575	10.707	9.864	43.238*
	[0.817]	[0.554]	[0.628]	[0.00002]
Number of observations (T)	144	626	144	626

Asymptotic t-statistics in parentheses and marginal significance level in brackets. Under the normality assumption, the skewness and excess kurtosis coefficients should be zero.

* Significant at the 1 percent level.

To provide a check on the methodology, a jump-diffusion process was estimated for the $/DM exchange rate over the fixed-rate period January 1959 to May 1971.[8] Over this 12-year period, there were two revaluations of the mark, which should be prime candidates for jumps. Indeed, as shown in Table 2, the χ_3^2 statistic of 266.7 amounts to a very strong rejection of the pure diffusion process. After the jump component is factored in,[9] the volatility of the remaining diffusion process drops dramatically, and the drift term becomes much smaller, which is consistent with small movements within the support points around the parity exchange rate. As expected, fixed-exchange-rate regimes are characterized by discontinuities that can be modeled by jump processes. The question now is whether such discontinuities also exist in flexible-exchange-rate regimes.

[8] The monthly data were taken from various issues of Pick's currency yearbook.

[9] The estimated jump intensity λ - 0.0625 is greater than what would be expected from two realignments In 148 months (λ - 2/148 = 0.013). This parameter is negatively correlated with θ, the drift of the log of the jump size, whose estimated value of 1.8 percent is less than the average size of a devaluation. As λ is overestimated and θ is underestimated, these parameters imply more frequent and smaller jumps than actually occurred. Although the jump process parameters are jointly significant, the imprecision in their estimated values can be attributed to the small number of jumps observed in the sample.

Table 2
Jump–diffusion process for the \$/DM rate for the fixed-rate period January 1959–April 1971

Process parameters						
Lognormal		Jump			Log-likeli-hood	Test of diffusion
μ (× 10²)	σ (× 10⁴)	λ	θ (× 10⁴)	δ^2 (× 10⁴)		χ^2_3
0.095	0.446*				531.35	
(1.74)	(8.60)					
−0.018	0.040*	0.0625*	0.0180	0.00033	664.71	266.7*
(−1.0)	(7.53)	(2.58)	(2.44)	(1.99)		

Diffusion process: $\ln(P_t/P_{t-1}) \sim N(\mu, \sigma^2)$. Jump process: intensity λ, jump size $\ln Y \sim N(\theta, \delta^2)$. Asymptotic t-statistics in parentheses. The German mark was revalued by 5 percent on March 2, 1961, and 9.29 percent on October 26, 1969. The χ^2 statistic tests the hypothesis of a pure diffusion process against a jump–diffusion model with constant parameters. Monthly observations ($T = 148$).

* Significant at the 1 percent level.

Tables 3 and 4 show the estimated coefficients for models (1) to (4) as well as tests of various hypotheses on the distribution of the logarithmic change in the \$/DM exchange rate and in the stock market index. A comparison between the stock market and the exchange rate provides interesting results. First, it should be noted that floating exchange rates. are typically less volatile than the stock market. Annualizing weekly variances by $\sqrt{52}$, for instance, the volatility of the mark is about 10.2 percent per annum, compared with 15.8 percent for the value-weighted stock market.

Turning now to the empirical fit of the various stochastic processes, a simple diffusion model seems to provide an adequate description of monthly stock returns: none of the x^2 tests are significant, and the SC criterion is minimized for the simple diffusion model. This is consistent with the findings of Fama (1976) and more recently of Jarrow and Rosenfeld (1984). For the monthly \$/DM rate, on the other hand, the hypothesis of a pure diffusion process is rejected against both the jump-diffusion and ARCH models: the marginal significance level of the respective χ^2_3 and χ^2_1 is about 2 percent for both alternatives. Similar conclusions hold for the χ^2_4 test of diffusion against combined ARCH and jump. These results must be qualified, however, given that the distributional assumptions are only valid asymptotically and that x^2 tests tend to reject too often in small samples. The SC criterion suggests that the ARCH model is a posteriori most probable, by a small margin, over the diffusion model. Overall, these results do not present overwhelming evidence against the diffusion model for monthly exchange rate movements.

The analysis of weekly data is shown in the lower parts of Tables 3 and 4. The χ^2_3 tests indicate that the jump-diffusion model is a significant improvement over the simple diffusion model in both the foreign exchange and stock markets. The results for the stock market are in contrast with those of Jarrow and Rosenfeld (1984), who reported no significant jump process for weekly stock market returns from 1962 to 1978. Further, Table 5 shows a summary of the χ^2_3 tests of no jump component for four weekly subperiods and three currencies. The subperiod analysis indicates that the jump process identified for the stock market is not spread evenly over the four subperiods. The jump process for the exchange rate, however, is significant for each of the four subperiods considered. Thus, the distribution of weekly exchange rate changes seems to be consistently characterized by discontinuities.

For weekly data, however, the normal distribution is also rejected against the alternative of a first-order ARCH process: the χ^2_1 statistic is highly significant for the \$/DM exchange rate and for the stock market. The economic significance of the ARCH process parameters can be illustrated as follows. If there were no movement in the \$/DM rate in the previous week, the conditional variance would be 0.0001456 per week, which translates into a weekly standard deviation of 1.2 percent, or annualizing by $\sqrt{52}$, an annual standard deviation of 8.7 percent. In contrast, if the previous weekly exchange rate movement were 3 percent-which is not an exceptional movement given the reported volatilities-the conditional variance would increase to $0.0001456 + 0.324(0.03)^2$, for an annual volatility of 15.1 percent, which is nearly twice the previous number! Clearly, the estimates of the ARCH process provided in Table 3 suggest economically important movements in exchange rate volatility. The figures for the stock market, on the other hand, yield a relatively less important ARCH effect: the same 3 percent movement in the stock market would increase the volatility from 14.1 to 17.1 percent per annum. It is also interesting to note that rejections of the null hypothesis of a constant variance are systematically weaker for the stock market than for the \$/DM rate.

Because both jump and ARCH components have been identified in weekly exchange rate movements-which was to be expected given the existence of fat tails-the question arises as to which of the two processes provides a superior description of the data. The values of the SC criterion in the right-hand columns of Tables 3 and 4 indicate that, even explicitly penalizing for its large number of parameters, the jump-diffusion process is a posteriori more probable than either the diffusion or the ARCH model for weekly data.

Further, given the evidence of heteroskedasticity in weekly data, one should test whether discontinuities still appear in the conditional distribution of returns. The fourth model in Tables 3 and 4 is a combined jump-ARCH process, which can be used to test this hypothesis. The χ^2_3 statistic tests the added significance of the jump process over and above the simple ARCH process. The values of the statistics are 75.76 and 45.34 for the \$/DM rate and the stock market index, respectively, which are highly significant. This suggests that discontinuities are present in the distribution of weekly exchange rates even after explicitly accounting for heteroskedasticity. These results could explain why Hsieh (1988) reports substantial leptokurtosis in the residuals of a more complex ARCH model, in which both the mean and variance of daily spot rate changes are allowed to vary over time. Similarly, Engle and Bollerslev (1986) also find that the residuals from a

Table 3
Stochastic processes for the \$/DM rate, January 1974–December 1985

Process parameters					
Lognormal		Jump			ARCH
μ (× 10²)	σ^2 (× 10⁴)	λ	θ (× 10⁴)	δ^2 (× 10⁴)	α_1
Monthly					
0.070	11.15*				
(0.251)	(8.48)				
−0.015	5.08*	0.40	0.21	15.56	
(−0.06)	(5.04)	(0.55)	(0.25)	(1.53)	
0.052	9.56*				0.126
(0.19)	(7.77)				(1.51)
0.039	5.14*	0.32	0.03	14.24	0.118
(0.15)	(4.81)	(0.39)	(0.02)	(1.21)	(1.40)
Weekly					
0.0134	2.052*				
(0.23)	(17.69)				
0.0531	0.173*	1.32	−0.022	1.38*	
(1.28)	(4.49)	(1.66)	(−0.52)	(6.25)	
0.0143	1.456*				0.324*
(0.26)	(14.73)				(4.37)
0.0567	0.088*	1.16	−0.014	1.23*	0.271*
(1.48)	(2.90)	(1.24)	(−0.24)	(5.05)	(3.96)

Diffusion process: $x_t \sim N(\mu, \sigma^2)$. Jump process: intensity λ, jump size ln $Y \sim N(\theta, \delta^2)$. ARCH process: $x_t \sim N(\mu, h_t)$, with $h_t = \sigma^2 + \alpha_1(x_{t-1} - \mu)^2$. Asymptotic t-statistics in parentheses; marginal significance levels in brackets. † denotes best model according to SC criterion, which allows us to compare nonnested models by explicitly penalizing for a larger number of parameters. The 1 percent critical levels for the χ^2_4, χ^2_3, and χ^2_1 are 13.3, 11.3, and 6.6, respectively. These statistics, respectively, test the hypothesis of a pure diffusion

Table 4
Stochastic processes for the CRSP value-weighted index January 1974–December 1985

Process parameters					
Lognormal		Jump			ARCH
μ (× 10²)	σ^2 (× 10⁴)	λ	θ (× 10⁴)	δ^2 (× 10⁴)	α_1
Monthly					
0.994*	21.46*				
(2.58)	(8.48)				
0.858	10.77*	0.51	0.27	21.3	
(2.37)	(4.84)	(0.36)	(0.26)	(1.31)	
1.134*	19.04*				0.107
(2.96)	(7.83)				(1.27)
0.993*	9.27*	0.65	0.06	15.7	0.089
(2.68)	(4.02)	(0.16)	(0.06)	(1.00)	(0.72)
Weekly					
0.228*	4.813*				
(2.60)	(17.69)				
0.280*	2.882*	0.17	−0.33	11.2	
(3.61)	(12.87)	(1.30)	(−0.71)	(2.40)	
0.255*	3.846*				0.199*
(3.11)	(15.50)				(3.50)
0.299*	2.459*	0.12	−0.45	12.3	0.162*
(3.94)	(11.78)	(1.54)	(−0.78)	(2.26)	(3.37)

See notes for Table 3.

Table 3
Extended

Log-likelihood	Test of diffusion model against: Mixed, χ^2_4	Jump, χ^2_3	ARCH, χ^2_1	Goodness of fit, χ^2_{19}	Schwarz criterion, SC
285.21				24.75 [0.17]	−560.5
290.15		9.88 [0.020]		18.76 [0.47]	−555.5
287.79			5.16 [0.023]	23.32 [0.23]	−560.7†
290.89	11.37 [0.023]	6.18 [0.103]			
1769.52				68.28* [0.0]	−3526.2
1813.42		87.80* [0.0]		14.08 [0.78]	−3594.6†
1792.08			45.12* [0.0]	60.49* [0.0]	−3564.8
1829.96	120.9* [0.0]	75.76* [0.0]			

process with constant parameters against a mixed jump–ARCH, a jump–diffusion, and an ARCH model. The degrees of freedom correspond to the additional number of parameters in the alternative hypothesis. The χ^2_3 statistics of 6.18 and 75.76 test the ARCH model against a mixed jump–ARCH model. Number of observations is 144 for monthly data and 626 for weekly data.

* Significant at the 1 percent level.

Table 4
Extended

Log-likelihood	Test of diffusion model significance: Mixed, χ^2_4	Jump, χ^2_3	ARCH, χ^2_1	Goodness of fit, χ^2_{19}	Schwarz criterion, SC
238.06				20.25 [0.38]	−466.2†
240.26		4.39 [0.22]		18.46 [0.49]	−455.7
239.36			2.60 [0.107]	14.51 [0.75]	−463.8
240.62	5.11 [0.276]	2.52 [0.472]			
1502.75				46.16* [0.0]	−2992.6
1529.06		52.60* [0.0]		32.44 [0.03]	−3025.9†
1517.12			28.74* [0.0]	45.29* [0.0]	−3014.9
1539.79	74.07* [0.0]	45.34* [0.0]			

Table 5
Comparative tests of stochastic processes for weekly subperiods January 4, 1974–December 31, 1985

Period (weekly)	T	\$/DM Jump, χ^2_3	\$/DM ARCH, χ^2_1	\$/yen Jump, χ^2_3	\$/yen ARCH, χ^2_1	\$/£ Jump, χ^2_3	\$/£ ARCH, χ^2_1	Value-weighted CRSP index Jump, χ^2_3	Value-weighted CRSP index ARCH, χ^2_1
Jan. 1974–	157	13.20*	5.90*†	108.9*†	22.01*	25.76*†	14.95*	16.06*	11.19*†
Dec. 1976		[0.0042]	[0.015]	[0.0]	[0.0]	[0.0]	[0.0001]	[0.0011]	[0.0008]
Jan. 1977–	156	35.87*	40.30*†	26.56*†	0.04	55.90*†	14.11*	17.72*†	2.32
Dec. 1979		[0.0]	[0.0]	[0.0]	[0.844]	[0.0]	[0.0002]	[0.0005]	[0.127]
Jan. 1980–	157	15.57*†	1.11	6.63	0.35	15.57*†	0.00	3.52	0.94
Dec. 1982		[0.0014]	[0.292]	[0.085]	[0.551]	[0.0014]	[0.997]	[0.318]	[0.333]
Jan. 1983–	154	14.54*	5.25†	46.11*†	5.12	23.91*	8.76*†	6.20	0.01
Dec. 1985		[0.0022]	[0.022]	[0.0]	[0.023]	[0.0]	[0.003]	[0.102]	[0.909]
Jan. 1974–	626	87.80*†	45.12*	119.8*†	12.53*	97.74*†	32.92*	52.60*†	28.74*
Dec. 1985		[0.0]	[0.0]	[0.0]	[0.0]	[0.0]	[0.0]	[0.0]	[0.0]

Numbers in brackets are marginal significance levels. The χ^2_3 statistics test the null of a diffusion process against the jump–diffusion process. The alternative hypothesis for the χ^2_1 statistics is the ARCH process. † indicates most likely model among diffusion, mixed jump–diffusion, or ARCH models, according to SC criterion. No † indicates that diffusion model is best.

* Significant at the 1 percent level.

generalized ARCH (GARCH) model display fatter tails than those expected from a normal distribution.[10]

The last task of this section is to ascertain whether the assumptions underlying the maximum-likelihood estimation are appropriate for this data set. This is important because the estimation technique relies heavily on a correct specification of the likelihood function. Tables 3 and 4 show goodness-of-fit test statistics for the first three models, obtained as follows. First, the observations $\{x_t\}$ were sorted in order of increasing magnitude and classified into $N = 20$ equally sized groups. Knowledge of the density function for each distribution allows us to compute a theoretical number of observations in each group. The goodness of fit between the actual and theoretical distributions is tested by summing the squares of the differences between the observed and theoretical number of outcomes in each group. Asymptotically, this test statistic has a χ^2_{N-1} distribution.[11] The predicted number of observations was computed from numerical integration of the relevant density functions over each interval.[12] Tables 3 and 4 show that the models fit well the data for monthly observations but that the diffusion and ARCH models are not quite appropriate for weekly data. The jump-

[10] The results in this article were also reproduced with a GARCH (1, 1) process, which actually provides a better fit than the simplest ARCH(1) model because it allows more persistence in the variances. The GARCH (1, 1) model was also rejected in favor of a more general alternative including jumps.

[11] The statistic $\sum_{j=1}^{N} (M_j - MP_j)^2/MP_j$, where M_j and MP_j are the observed and predicted number of outcomes, respectively, has an asymptotic distribution which is actually bounded between a χ^2_{N-1} and a χ^2_{N-1-k}, where k is the number of parameters estimated by maximum likelihood. See, for instance, Kendall and Stuart (1967). A conservative test was used here, given that the distribution is only valid asymptotically.

[12] For the ARCH process, innovations divided by the conditional volatility should follow a standard normal distribution.

diffusion process, on the other hand, does not seem incorrectly specified, which supports the general conclusions of this section.

4. Implications for Option Pricing

The finding of marked jump processes in exchange rates has important implications for the pricing of currency options. Recently, Bodurtha and Courtadon (1987) tested the ability of the American optionvaluation model to explain the pricing of currency options quoted on the Philadelphia Stock Exchange. Focusing on the relative pricing error, the model seems to underprice short-term out-of-the-money options by as much as 29 percent. At-the-money and in-the-money options are generally slightly overpriced, with the bias most pronounced for short-maturity options.[13] These results are in contrast with what has been found generally for stock options.[14]

As suggested by Bodurtha and Courtadon (1987), the directions of these biases are generally consistent with a mixed jump-diffusion process. Consider the price of an out-of-the-money call option close to maturity. If the exchange rate follows a diffusion process, the chance of exercising the option at maturity may be quite small; with a jump process, however, one jump may be sufficient to move the option in the money, which implies that a diffusion model will underprice the option.[15] Thus the issue is whether the empirically observed large biases can be fully accounted for by a jump process, which is the subject of this section.

Consider the sensitivity of the European[16] call option valuation model to the introduction of a mixed jump-diffusion process for exchange rates. The Black-Scholes formula assumes a lognormal diffusion process; modified for foreign currency options,[17] it can be written as

$$F_e(S, \tau; r, r^*, \sigma^2, K) \equiv e^{-r^*\tau}C(S, \tau; r - r^*, \sigma^2, K) \tag{5}$$

where C is the usual Black-Scholes pricing formula for call options, S is the spot rate expressed in dollars per unit of the foreign currency, τ is the time to expiration on a per annum basis, r and r^* are the domestic and foreign rates of interest, σ^2 is the annual variance, and K is the strike price. The foreign rate of interest can be interpreted as the continuous dividend

[13] Similar results are reported by Borensztein and Dooley (1987) and Hsieh and Manas-Anton (1989), the latter for options on mark *futures.*

[14] McBeth and Merville (1980), for instance, find that out-of-the-money options ate overpriced by the Black-Scholes model, while in-the-money options seem mostly underpriced.

[15] Similarly, deep in-the-money options could also appear too expensive relative to a diffusion model: the "insurance" feature of the option, virtually worthless with a diffusion process, should be greater when jumps are present.

[16] As Shastri and Tandon (1986) showed, the difference in the valuation of American and European options is small for most all options, especially in cases where the foreign interest rate is lower than the U.S. interest rate, which is typical of the German mark. For instance, Jorion and Stoughton (1989) analyzed the value of the early-exercise premium from market data and found that it averages 2 percent for $/DM call options.

[17] See, for instance, Carman and Kohlhagen (1983).

yield on the underlying asset. With an added jump component, Merton's (1976) valuation model[18] can be extended to the case of currency options as

$$F(S, \tau; r, r^*, \sigma_0^2, \lambda, \theta, \delta^2, K)$$
$$\equiv e^{-r^*\tau} \sum_{j=0}^{\infty} \frac{e^{-\lambda T e^{\theta+\delta^2/2}}(\lambda T e^{\theta+\delta^2/2})^j}{j!}$$
$$\cdot C\left(S, \tau; r - r^* + j\frac{\theta + \delta^2/2}{\tau} - \lambda(e^{\theta+\delta^2/2} - 1), \sigma_0^2 + j\frac{\delta^2}{\tau}, K\right) \quad (6)$$

where σ_0^2 is now the variance of the pure diffusion process and λ, θ, and δ^2 are the parameters of the jump process, defined previously. An investor ignoring the jump component would estimate the total variance of an assumed diffusion process as $\sigma^2 = \sigma_0^2 + \lambda\delta^2$, on which the diffusion option price F_e will be based. For example, from the weekly data in Tables 3 and 4, the annual volatility of the \$/DM rate would be 10.2 percent, against a volatility of 15.8 percent for the value-weighted stock market.

The extent of the mispricing can be measured by the relative difference[19] $\epsilon = (F - F_e)/F$, using the parameters previously estimated from weekly data over the period 1974 to 1987. F_e is computed from Equation (5) with $\sigma^2 = \sigma_0^2 + \lambda\delta^2$, and F is computed from Equation (6) using the same values of σ_0^2, λ, and δ^2. The analysis is performed for \$/DM currency call options and hypothetical call options on the value-weighted stock market.

Tables 6 and 7 show relative and absolute mispricing errors for typical option parameters. The options have been classified by time to maturity and by the ratio S/K, taken to represent in-, at-, and out-of-the-money classes. All prices are scaled by the strike price. For stock options, the out-of-the-money option is defined here by a spot price level of 0.95. For currency options, a different definition of the out-of-the-money options was used to account for the lower volatility of exchange rates. Specifically, the amount by which the stock option was out of the money (0.05) was multiplied by the ratio of the volatilities to yield $0.05 \times 10.2/15.8 = 0.0322$. Thus, the out-of-the-money currency option was taken as the one for which the exchange rate was 0.9678. With this adjustment, the probability of ending in the money is the same for the currency and stock options, based on a simple diffusion process. In the same fashion, in-the-money options are defined by a spot price of 1.05 and 1.0322 for stock options and currency options, respectively. The other parameters were fixed at r = 5 percent and r^* = 5 percent for the dollar and foreign interest rates.

Some noticeable differences appear in the mispricing of currency and stock options. Using the estimated parameters, the Black-Scholes model

[18] See also Jarrow and Rudd (1983). The Merton model relies on the assumption that the jump component is diversifiable and therefore not priced. This allows the risk of the jump component to be eliminated through a risk-neutral argument.

[19] This definition is consistent with that of Bodurtha and Courtadon (1987) and thus an be used for comparing numerical results.

Table 6
Hypothetical options: relative pricing differences in percent [$\epsilon = (F - F_0)/F$; parameters: $K = 1$, $r = 5\%$, $r^* = 5\%$]

	Months to maturity							
	0.50	0.75	1	1.5	3	4.5	6	9
$/DM options								
Out-of-the-money	17.15	4.63	0.72	−1.23	−0.99	−0.68	−0.50	−0.32
(S = 0.9678)	(0.86)	(0.49)	(0.15)	(−1.14)	(−1.43)	(−1.53)	(−1.60)	(−1.67)
At-the-money	−4.62	−2.78	−1.98	−0.93	−0.61	−0.40	−0.29	−0.19
(S = 1.00)	(−1.52)	(−1.58)	(−1.66)	(−1.78)	(−1.76)	(−1.70)	(−1.62)	(−1.43)
In-the-money	0.31	0.23	0.15	−0.05	−0.09	−0.10	−0.09	−0.07
(S = 1.0322)	(2.12)	(1.34)	(0.69)	(−0.24)	(−0.54)	(−0.66)	(−0.68)	(−0.67)
Stock index options								
Out-of-the-money	12.81	−0.87	−4.46	−4.50	−2.95	−1.61	−0.98	−0.49
(S = 0.95)	(0.72)	(−0.06)	(−0.41)	(−0.80)	(−0.90)	(−1.01)	(−1.18)	(−1.18)
At-the-money	−7.37	−6.12	−5.26	−3.09	−1.86	−0.91	−0.50	−0.21
(S = 1.00)	(−1.41)	(−1.20)	(−1.10)	(−0.98)	(−0.91)	(−0.83)	(−0.73)	(−0.44)
In-the-money	−0.79	−1.19	−1.40	−1.28	−0.84	−0.37	−0.16	−0.02
(S = 1.05)	(−0.37)	(−0.49)	(−0.56)	(−0.68)	(−0.65)	(−0.49)	(−0.30)	(−0.06)

Asymptotics t-statistics in parentheses. Stochastic process parameters taken from Tables 3 and 4, weekly data. Error measured as "true" jump–diffusion price minus Black–Scholes price divided by true price: $(F - F_0)/F$, where $F = F(S, \tau; r, r^*, \sigma_0^2, \lambda, \theta, \delta^2, K)$, $F_0 = F_0(S, \tau; r, r^*, \sigma^2, K)$, and $\sigma^2 = \sigma_0^2 + \lambda\delta^2$.

underprices short-term out-of-the-money currency options by about 17 percent, which can partly explain the 29 percent underpricing reported by Bodurtha and Courtadon (1987). This result can be traced to the ratio $\lambda\delta^2/(\lambda\delta^2 + \sigma_0^2)$, which represents the fraction of the variance caused by the jump component; this ratio is 96 percent for the $/DM rate, much higher than the 36 percent figure for the stock market.[20] As a result, the diffusion process underestimates the likelihood of a jump that would bring one of those out-of-the-money short-lived DM call options into the money. The mispricing is weaker for stock market options, which is in line with the interpretation of Ball and Torous (1985), who report few operational discrepancies between the two option valuation models. Finally, the extent of the mispricing decreases as the time to maturity increases, as observed empirically.

One shortcoming of this analysis is that it does not take into account the estimation error in the process parameters. Given the invariance property of maximum-likelihood estimators, these reported pricing errors are also maximum-likelihood estimates. But it would also be interesting to construct asymptotic standard errors for the reported point estimates. These could be used to test whether the reported pricing errors are significantly different from zero given the sampling variation in the data and the dependence of the pricing errors on the estimated parameters. As Table 6 shows, the results from the asymptotic t-statistics are generally inconclusive for the relative pricing errors. However, the absolute pricing discrepancy for the shortest-term out-of-the-money currency option has a t-value of 1.8.

[20] Holding this ratio constant, the mispricing would be even more important for a process characterized by large jumps that occur infrequently; that is, for a smaller λ and larger δ^2.

Table 7
Hypothetical options: absolute pricing differences in cents [$\epsilon = F - F_J$ parameters: $K = 1$, $r = 5\%$, $r^* = 5\%$]

	Months to maturity							
	0.50	0.75	1	1.5	3	4.5	6	9
$/DM options								
Out-of-the-money	1.05	0.57	0.14	−0.61	−0.76	−0.78	−0.74	−0.64
($S = 0.9678$)	(1.80)	(0.67)	(0.16)	(−1.04)	(−1.80)	(−2.36)	(−2.72)	(−3.23)
At-the-money	−3.66	−2.75	−2.27	−1.52	−1.21	−0.97	−0.82	−0.65
($S = 1.00$)	(−2.32)	(−2.40)	(−2.57)	(−2.78)	(−2.69)	(−2.48)	(−2.25)	(−1.83)
In-the-money	1.03	0.84	0.53	−0.18	−0.37	−0.43	−0.43	−0.39
($S = 1.0322$)	(2.11)	(1.80)	(0.71)	(−0.24)	(−0.54)	(−0.67)	(−0.69)	(−0.64)
Stock index options								
Out-of-the-money	1.10	−0.15	−1.24	−3.26	−3.39	−2.77	−2.17	−1.49
($S = 0.95$)	(0.70)	(−0.06)	(−0.42)	(−0.84)	(−0.97)	(−1.13)	(−1.37)	(−1.32)
At-the-money	−8.81	−9.07	−9.06	−7.64	−5.68	−3.44	−2.16	−1.10
($S = 1.00$)	(−1.47)	(−1.25)	(−1.15)	(−1.03)	(−0.97)	(−0.88)	(−0.78)	(−0.45)
In-the-money	−4.00	−6.10	−7.33	−7.37	−5.20	−2.55	−1.15	−0.20
($S = 1.05$)	(−0.38)	(−0.49)	(−0.57)	(−0.69)	(−0.66)	(−0.50)	(−0.31)	(−0.06)

See notes to Table 6.

Thus, there is some evidence that the reported underpricing of short-term out-of-the-money currency options is not due to sampling variation.

At-the-money call options appear overpriced in both markets, and significantly so for currency options. Pricing errors for options in the money seem relatively small. Finally, the extent of the mispricing decreases with the time to maturity. Most of these results are consistent with empirical observations in the currency options market.

In summary, the estimates of mixed jump-diffusion processes reveal that ignoring the jump component in exchange rates can lead to serious mispricing errors for currency options. This can account in large part for the large pricing errors reported in previous empirical tests of option pricing models in the foreign currency options market. On the other hand, smaller discrepancies were found in stock market options, which can be explained by the fact that discontinuities are harder to identify in the stock market index.

5. Conclusions

This article has investigated the existence of discontinuities in the sample path of exchange rates and of a value-weighted U.S. stock market index. It was found that exchange rates display significant jump components, which are more manifest than in the stock market. These discontinuities seem to arise even after explicit allowance is made for possible heteroskedasticity in the usual diffusion process and appear very strongly in weekly data but less so in monthly data. The statistical analysis was performed side by side for the foreign exchange market and the stock market, and it suggests important differences in the structure of these markets.

The economic importance of this result was illustrated for the currency options market, for which it was shown that ignoring the jump component can induce serious mispricing of currency options. Previous models of currency options have always relied on the assumption of continuous sample paths for exchange rates. Using the estimated parameters, numerical examples showed that about two-thirds of the 29 percent biases reported for short-term out-of-the-money options can be explained by a mixed jump-diffusion process. Consequently, successful models of short-term movements in exchange rates should be consistent with these empirical findings.

Appendix: Maximum-Likelihood Estimation

This Appendix briefly summarizes the maximum-likelihood estimation method used in the article. If prices follow a diffusion process with constant drift parameter $E[\Delta P/P] = \alpha$ and constant variance $V[\Delta P/P] = \sigma^2$, logarithm of price relatives $x_t \equiv \ln(P_t/P_{t-1})$ is normally distributed with mean $\mu \equiv \alpha - \sigma^2/$ and variance σ^2, With T independent observations, the logarithm of the likelihood function L $(\phi; x)$, viewed as a function of the parameter vector $\phi = \mu, \sigma^2)$, can be written as

$$l_N = -\frac{T}{2}\ln(2\pi) + \sum_{t=1}^{T} \ln\left[\frac{1}{\sqrt{\sigma^2}} \exp\left(\frac{-(x_t-\mu)^2}{2\sigma^2}\right)\right] \tag{A1}$$

Consider now a Poisson process where λ is the mean number of jumps occurring per unit time and where the jump size Y has a posited distribution $\ln Y \sim N(\theta, \delta^2)$. The log-likelihood function for the mixed jump-diffusion process is

$$l_J = -T\lambda - \frac{T}{2}\ln(2\pi) + \sum_{t=1}^{T} \ln\left[\sum_{j=0}^{\infty} \frac{\lambda^j}{j!} \frac{1}{\sqrt{\sigma^2 + \delta^2 j}} \exp\left(\frac{-(x_t - \mu - \theta j)^2}{2(\sigma^2 + \delta^2 j)}\right)\right] \tag{A2}$$

In order to numerically optimize the above function, the infinite sum has to be truncated after some value of *N*. Ball and Torous (1985) derived a formula for an upper bound on the truncation error, which could be used to select a desirable *N*. In practice, truncation at $N = 10$ provides satisfactory accuracy for all parameter values encountered in this article.

Leptokurtic distributions can also arise because of time-varying parameters instead of discontinuities. One such model is an ARCH process introduced by Engle (1982). This is a tractable specification where the conditional variance h_t is explicitly modeled as a nonstochastic function of past squared innovations. Because economic theory has little to say about the appropriate number of lags to include, the simplest specification chosen here is the first-order ARCH model, where the conditional volatility can be written as

$$h_t = \alpha_0 + \alpha_1\epsilon_t^2 = \alpha_0 + \alpha_1(x_{t-1} - \mu)^2 \tag{A3}$$

with ϵ_t defined as $x_t - \mu$. In the absence of heteroskedasticity, the parameter

α_1 should be zero, in which case α_0 represents the variance of a stationary diffusion process. This could be expanded to include a weighted average of a number of past observations, or in general any predetermined information. The log-likelihood function for this ARCH model is

$$l_A = -\frac{T}{2}\ln(2\pi) + \sum_{t=1}^{T} \ln\left[\frac{1}{\sqrt{b_t}}\exp\left(\frac{-(x_t-\mu)^2}{2b_t}\right)\right] \tag{A4}$$

If the conditional distribution of $\in$ has a Poisson distribution, the log-likelihood function can be written as

$$l_{\mathcal{N}} = -T\lambda - \frac{T}{2}\ln(2\pi) + \sum_{t=1}^{T} \ln\left[\sum_{j=0}^{\infty} \frac{\lambda^j}{j!}\frac{1}{\sqrt{b_t+\delta^2 j}}\exp\left(\frac{-(x_t-\mu-\theta j)^2}{2(b_t+\delta^2 j)}\right)\right] \tag{A5}$$

This likelihood function includes the jump process, the ARCH process, and the normal process as special cases. Therefore it can be used to construct a generalized likelihood ratio $\Lambda = \sup_{\phi\in\Phi_0} L(\phi; x)/\sup_{\phi\in\Phi} L(\phi; x)$, where the likelihood functions have been maximized (1) over the parameter space $\phi \in \Phi_0$, under the null hypothesis and (2) over the parameter space $\phi \in \Phi = \Phi_0 \cup \Phi_1$, which includes the alternative Φ_1. Under the null hypothesis, the statistic $-2\ln\Lambda$ has a chi-square distribution with degrees of freedom equal to the number of parameters between the two models. This asymptotic result holds because the two hypotheses Φ_0 and Φ are nested in the parameter space.

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